

Brane -Antibrane Forces

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ABSTRACT

The force between like sign BPS saturated objects generally vanishes. This is a reflection of the fact that BPS states are really massless uncharged particles with nonvanishing momenta in compactified directions. Two like sign BPS objects with zero relative velocity can be viewed as a boosted state of two neutral massless particles in a state of vanishing relative motion. By contrast two unlike sign BPS particles may be thought of as colliding objects moving in opposite directions in compact space. This leads to complicated interactions which are totally intractable at present. We illustrate this by considering the potential between opposite sign zero-D- branes .

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1 Introduction

Perturbative string theory as presently formulated is completely inadequate for the study of planckian and transplanckian processes. No matter how small the dimensionless string coupling λ is, the perturbation expansion breaks down at the planck energy. An apparent exception to this rule is presented by BPS saturated states, some of whose properties are exactly known. Although this is true, there is a certain sense in which BPS physics is often really just a reflection of zero energy physics. In particular, consider those objects which may (possibly after some duality transformation) be thought of as Kaluza Klein charges of minimal mass. The charge of these objects is just their momenta in the compact dimensions. Consider two such BPS particles, each at rest in a common rest frame. Suppose that their charges are all proportional to their masses. In this case their momentum vectors (including the compact components) are parallel. They can be thought of as massless neutral particles with vanishing center of mass energy. This is the class of BPS configurations for which the forces exactly cancel. A similar pair of unlike charges correspond to a pair of oppositely moving particles which collide with center of mass energy equal to

$$E_{cm} = 2|Q_1 Q_2| \tag{1}$$

where Q_i is the charge of particle i . These particles are expected to undergo violent interactions, perhaps leading to black hole formation, as the charges become large. In this paper we consider the BPS particle states (zero branes) that have been described by Polchinski³ [1] as D-branes. We show that the interactions between oppositely charged D-branes are quite different from those between BPS states. In particular, although one might have thought that one could describe the dynamics by a Born Oppenheimer potential until momentum transfers become of order the zero brane mass, we show that the potential becomes complex and the Born-Oppenheimer approximation breaks down when the distance between the branes is of order the string scale.

2 Forces Between Branes

We consider weakly coupled ten dimensional type 2a strings. This theory admits zero-branes which carry Ramond-Ramond charge. These charges can be thought of as Kaluza Klein charges in the 11 dimensional M-theory which reduces to type 2a under compactification on a circle. Therefore the remarks of the introduction apply to these objects. In particular two like sign branes at rest should not interact. In fact Polchinski has calculated the force between such particles and finds that it vanishes. Let us briefly review the argument.

The mass of a zero-brane is of order $\frac{1}{\lambda}$ where λ is the type 2a string coupling constant. Therefore when $\lambda \rightarrow 0$ the mass tends to infinity and the particles can be treated

³ Polchinski's work was motivated by earlier work of Green [2] who first made the crucial observation that Dirichlet states in the superstring preserve half the supersymmetries.

nonrelativistically. The calculation of the force between two like sign particles separated by distance Y is computed in lowest order as an open string one loop annulus diagram with the boundary conditions that the two ends of the string are located at the sites of the two charges. Polchinski finds the result that the potential is proportional to

$$A = \int \frac{dt}{t} (2\pi t)^{-1/2} e^{-tY^2/8\pi^2\alpha'^2} \prod_{n=1}^{\infty} (1 - q^{2n})^{-8} \\ \frac{1}{2} \left\{ -16 \prod_{n=1}^{\infty} (1 + q^{2n})^8 + q^{-1} \prod_{n=1}^{\infty} (1 + q^{2n-1})^8 - q^{-1} \prod_{n=1}^{\infty} (1 - q^{2n-1})^8 \right\} \quad (2)$$

where $q = e^{-t/4\alpha'}$ and the integration variable t is the proper time in the open string channel. The first two terms in the large curly bracket correspond to the exchange of NS-NS closed strings and the third term to R-R closed strings. As pointed out by Polchinski, the NS-NS terms cancel the R-R terms by the "usual abstruse identity".

Now consider the effect of replacing one of the charges by an opposite sign charge. The only effect is to change the sign of the R-R term. The cancellation no longer occurs and the potential is proportional to

$$A = \int \frac{dt}{t} (2\pi t)^{-3/2} e^{-\frac{t}{4\alpha'}(\frac{Y^2}{2\pi^2}-1)} f(t) \quad (3)$$

where $f(t)$ is a function which tends to 1 for large t and rapidly tends to 0 as $t \rightarrow 0$.

From this equation we first of all see that the potential no longer vanishes. More interesting is the fact that the integral diverges [2] for $Y^2 < 2\pi^2$. To see the nature of the singularity we can differentiate with respect to Y in order to remove the dependence on the behavior of f near $t = 0$. The resulting integral defines the force and is given proportional to

$$F = Y \int \frac{dt}{t} (2\pi t)^{-1/2} e^{-\frac{t}{4\alpha'}(\frac{Y^2}{2\pi^2}-1)} f(t) \quad (4)$$

Let us define $Z = \frac{1}{4\alpha'} \left\{ \frac{Y^2}{2\pi^2} - 1 \right\}$ and rescale the integral to get

$$F = \frac{1}{\sqrt{Z}} Y \int \frac{du}{u^2} e^{-u} f\left(\frac{u}{Z}\right) \quad (5)$$

For $Z \rightarrow 0$ the function f tends to 1 and the integral becomes

$$F = \frac{1}{\sqrt{Z}} Y \int \frac{du}{u^2} e^{-u} \propto \frac{1}{\sqrt{Z}} Y \quad (6)$$

Thus we see that the attractive force diverges as $Z \rightarrow 0$ and its analytic continuation to $Y^2 < 2\pi^2$ is complex. The potential itself is finite at $Z = 0$ but is also complex for $Y^2 < 2\pi^2$.

3 Physical Interpretation

A potential becoming complex is an indication that inelastic channels are opening up. This is not too surprising since as the opposite sign charges approach they can annihilate into fundamental strings. It is revealing that the inelasticity sets in at a distance of order $\sqrt{\alpha'}$. This seems to support the view that D-branes carry a string scale halo around them, even though they appear pointlike.

To get further insight we note that the quantity Z is proportional to the squared energy of the lightest open string connecting the two oppositely charged zero-branes. When Z becomes negative a tachyon develops. The situation is very similar to the thermal tachyon which occurs at the hagedorn temperature. In that case a condensate of winding states forms when the periodic imaginary time becomes too small. In the present case the tachyon forms when the two charges get too close. To follow the system beyond this point we must compute higher order terms in the tachyon effective potential. These can be computed in tree approximation by calculating multi tachyon vertex operators. It is easy to see that the effective potential must only contain terms even in the tachyon field T . An open string can be characterized by which charges its ends are attached to. If we call the positive charge A and the negative charge B then we can classify the open strings as AA , BB , AB , and BA . Any transition conserves the number of A ends modulo 2. Likewise for B ends. We are interested in the potential governing the AB and BA strings. The ground state string of lowest energy will actually be a symmetrized sum of these two configurations. The discrete symmetries can be seen to imply a Z_2 symmetry for the effective potential for this ground state string. The quadratic term is just the mass term proportional to Z . The next term is quartic and of order g^2 , where $g = \sqrt{\lambda}$ is the open string coupling. Thus the potential has the form

$$V = ZT^2 + Cg^2T^4 \quad (7)$$

where the constant C is computed from the four point tachyon scattering amplitude.

When Z becomes negative the potential becomes negative and the field rolls to its minimum at $T \sim 1/g$. This implies a condensate containing about $1/g^2$ open strings. If the constant C is positive then the transition is second order and the tachyon field is continuous. In fact we find that C is negative. This implies that as Z decreases a region of metastability occurs and that the condensate is discontinuous. We can not systematically follow the system into this region but presumably the final state of the evolution is the annihilation of the pair into ordinary closed strings. Indeed, the process should be quite similar to the annihilation of baryon and antibaryon in large N QCD. The description of condensate formation is also temptingly similar to the formation of a thermal condensate near the horizon of an ordinary

black hole. It is likely that the phenomenon that we are seeing is just the onset or precursor of non-extreme black hole formation.

At higher energies, black holes have a large degeneracy that we identify with their thermodynamic entropy. One may wonder if any sign of degeneracy can be found in the brane-pair system. There is one obvious source. Due to the symmetry with respect to changing the sign of T the condensate can form with either positive or negative sign. This degeneracy should be lifted by tunnelling but the tunnelling rate is exponentially small as $g \rightarrow 0$. Thus the system has two, essentially degenerate, states for weak coupling. In other words, for all practical purposes, as we allow the branes to adiabatically approach one another, an instability forms and the condensate can fall either way. This appears to be a source of macroscopic irreversibility.

We suspect that the phenomenon we have discovered may be continuously connected to black hole formation processes and that the single bit of entropy in the two brane system will evolve into the Bekenstein-Hawking entropy as the mass of the branes increases. We could try to test this by studying a system of M coincident branes and N coincident anti-branes separated by a distance Y , in the limit that M and N get large. The open string tachyon field then becomes an $M \times N$ matrix and one can imagine that there will be a large entropy associated with different stable condensates. However, we believe that the single closed string exchange graph on which our calculation is based does not contain the nonlinear gravitational physics which surely becomes important for large mass. Thus although the process we have studied may indeed be continuously connected to black hole formation, our approximation is likely to break down before we can test this conjecture.

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